Y10 MYP Maths (Mr Davis' Group)

A bit more on cubics

Re-cap

Recently, we learned how to completely factorise a cubic, given one of the linear factors.
Here's another example:

Factorise $2x^3 + 3x^2 - 2x - 3$, given that $x + 1$ is a factor

There are several methods available to us, but the one we chose recently was algebraic division:

$$
\begin{array}{r|l}
 & 2x^2 + x - 3 \\
\hline
(x + 1) & 2x^3 + 3x^2 - 2x - 3 \\
 & \underline{2x^3 + 2x^2} \\
 & \qquad\; x^2 - 2x \\
 & \qquad\; \underline{x^2 + x} \\
 & \qquad\qquad -3x - 3 \\
 & \qquad\qquad \underline{-3x - 3} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

Thus, $2x^3 + 3x^2 - 2x - 3$ can be written as $(x + 1)(2x^2 + x - 3)$

In fact the quadratic in the 2nd bracket can also be factorized to give:

$$2x^3 + 3x^2 - 2x - 3 \equiv (x + 1)(2x + 3)(x - 1)$$

An important question to consider – what if we hadn't been told of the linear factor at the beginning?

Again, there are several ways of handling this. One of them (that's not too difficult) is to trial different factors (and so different algebraic divisions). A little common sense goes a long way, here!

For example, factorise $x^3 - 3x^2 + 4x - 2$

Remember, when we have our answer, the pure numbers in the factors have to multiply out to give -2. There aren't too many possibilities to consider.

Let's see if $(x + 1)$ is a factor. In other words, let's try dividing $x^3 - 3x^2 + 4x - 2$ by $(x + 1)$, and seeing if there's no remainder. Try it (it doesn't work!) Given that it doesn't work, try dividing by $(x - 1)$ instead. This **does** work!

$$
\begin{array}{r|l}
 & x^2 - 2x + 2 \\
\hline
(x - 1) & x^3 - 3x^2 + 4x - 2 \\
 & \underline{x^3 - x^2} \\
 & \quad -2x^2 + 4x \\
 & \quad -2x^2 + 2x \\
 & \qquad\qquad 2x - 2 \\
 & \qquad\qquad \underline{2x - 2} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

Thus,

$$x^3 - 3x^2 + 4x - 2 \equiv (x - 1)(x^2 - 2x + 2)$$

(The quadratic factor cannot be further factorized)

A. Factorise the following (they all have at least one linear factor):

(a) $x^3 - 6x^2 + 5x + 12$ **(b)** $x^3 - 2x^2 - 5x + 6$ **(c)** $x^3 + 7x^2 - 4x - 28$

B. Solve the following (where possible):

(a) $x^3 + 4x^2 - x - 4 = 0$ **(b)** $x^3 - 3x^2 - 13x + 15 = 0$ **(c)** $x^3 - 4x - 3 = 0$